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A Review of Model Order Reduction Techniques for Linear Systems

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Abstract—*Model order reduction techniques play a very important role for modeling of complex system. A vast literature is available in Control related journals for model order reduction techniques for linear system. Although it is a herculean task to review all the model order reduction techniques developed so far. However, an attempt has been made to review some important techniques for model order reduction of a linear system in time domain and in frequency domain.*

1. INTRODUCTION

The various methods discussed in this paper for reduction of linear systems are based on Aggregation method, Davison technique of model reduction, Krishnamurthy and V. Seshadri approach of model order reduction, Truncation method of Reduction, Pade approximation method, Moment matching method, Reduction of transfer functions by the stability equation, M. F. Hutton and B. Friedland approach of model reduction, S. Mukherjee and R. N. Mishra approach on model order reduction of linear systems using Error minimization technique, Jayant Pal and L. M. Ray approach of model reduction for multivariable system, Model order reduction by Differentiation method, Cauer Methods, Stability equation method and Modified Cauer Continued Fraction approach, reduced Stable -order Pade approximants using the Routh-Hurwitz array reduced order modelling of linear dynamic systems using Eigen spectrum analysis and modified Cauer Continued fraction.

The remaining part of this paper is organized as follows:

The mathematical formulation of the problem in frequency domain and in time domain is discussed in section 2. finally, conclusion and references are presented.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Representation of model order reduction [18,19]:

Frequency domain representation:

The transfer function of a higher order SISO system can be expressed as

$$G(s) = \frac{a_0 + a_1 s + a_2 s + \ldots + a_m s^m}{b_0 + b_1 s + b_2 s + \ldots + b_n s^n}; \quad m £ n$$

Suppose, there is need to determine reduced order model of above higher order system and if $'r'$ be the order of reduced order model, then reduced order model of above system

Can be written as

$$G_r(s) = \frac{c_0 + c_1 s + c_2 s + \ldots + c_q s^q}{d_0 + d_1 s + d_2 s + \ldots + d_r s^r}; q £ r$$

The main purpose is to determine the coefficients $c_0, c_1, \ldots, c_q$ and $d_0, d_1, \ldots, d_r$ such that the response of the original model $G(s)$ is equivalent to $G_r(s)$.

Time domain representation:

Let us consider the higher-order linear system which is expressed in state space form as:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

In above equation, state matrix, input matrix and output matrix are represented by $x(t)$, $u(t)$, and $y(t)$, respectively. These matrices can be written as

$$x(t) = [x_1, x_2, \ldots, x_n] \; u(t) = [u_1, u_2, \ldots, u_n]$$

$$y(t) = [y_1, y_2, \ldots, y_n]$$

In this equation, A is a system matrix of the order $(n \times n)$, B is an input matrix of the order $(n \times m)$, C is an output matrix of the order $(p \times n)$, and D is a transmission matrix of the order $(p \times m)$.

A reduced order model of the original system can be assumed in the form:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t)$$
$$y_r(t) = C_r x(t) + D_r u(t)$$

Where, $x_r(t),\ y_r(t),\ A_r, B_r, C_r, D_r$ are corresponding matrices of suitable reduced dimensions. Suppose *n* be the order of a higher-order system and *r* be the order of reduced-order system.

The system presented in time domain can also be represented in frequency domain. Let us consider a higher-order system transfer function which is represented by $G(s)$. This is given as

$$G(s) = C[sI - A]^{-1} B + D$$

Similarly for the reduced order model, we can represent the transfer function by

$$G_r(s) = C_r[sI - A_r]^{-1} B_r + D_r$$

The main idea of model order reduction is that the error between original model and reduced model must be very low or in the prescribed error band. This can be shown as follows:

$$\left\| G(s) - G_r(s) \right\| < \varepsilon$$

where ε is the tolerance between both the models.

2.1. Aggregation method[1]:

This technique has been proposed by Aoki in 1968. This aggregation term is appeared in the economic literature before arriving in control literature. Here, the model is derived by aggregating the original system state vector into a lower-dimension vector.

2.2 Davison Technique of model reduction 2]:

This technique has been proposed in 1966 by E. J. Davison. This method is based on dominant eigenvalue approach. The idea of this method is to neglect the eigenvalues of the original system which are farthest away from the origin and retain only dominant eigenvalues. This method is applicable to any large, linear, and multivariable system and but it is time consuming for higher-order systems.

2.3 V. Krishnamurthy and V. Seshadri approach of model order reduction [3]:

This method of model order reduction is based on Routh criterion. In this method, the technique which is applicable to check the stability of the system, the same is applicable to reduce the model of the system. In this approach, reduced order model is determined by applying Routh criterion to both denominator and numerator polynomial. This method is simple and also applicable to unstable system

2.4. Truncation method of Reduction [4]:

This method has been suggested by Y. Shamash. This is the simplest method of model order reduction. In this method, reduced order model can be determined by truncating higher order terms, particularly for lower order system. However, for higher order system, reduced order model will be unstable if original reduced order model is stable.

2.5. Pade approximation method [5]:

This approach is based on matching a few coefficients of the power series expansion about $s = 0$, of the reduced-order model with the corresponding coefficients of the original model. This approach was first proposed by Y. Shamash. This is given below.

Consider a function as,

$$f(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + \ldots$$

and a rational function $U_m(s)/V_n(s)$, , where, $U_m(s)$ and $V_n(s)$ are the m^{th} and n^{th} order polynomials in s respectively, and $m \leq n$. The rational function $U_m(s)/V_n(s)$, is said to be a Padé approximant of $f(s)$ if and only if the first *(m+n)* terms of the power series expansion of $f(s)$ and $U_m(s)/V_n(s)$, are identical.

2.6. Moment matching method of model reduction [6]:

This method is based on matching the time-moments of the full model's impulse response to those of the reduced model.

2.7. Reduction of transfer functions by the stability equation method: [7]

This method has been proposed by T.C. Chen and C. Y Chang. This method is based on forming stability equations of numerator and denominator polynomials of high-order system. Further, non-dominant poles and zeros will be discarded. This approach preserves the stability of the reduced order models.

2.8. M. F. Hutton and B. Friedland approach of model reduction (1975)[8]:

This method has come in the literature in 1975. This method is based on the concept of Routh criterion. It is called as Routh approximation method. The basic idea of this method is to develop Routh array for the original system and then construct the approximant in such a manner that the coefficients of the Routh array agree up to a given order. In this method if the original system is stable, then reduced order system will also stable.

2.9. T. N. Lucas approach of model reduction using Factor division [9]:

This method is useful for model reduction. The main advantage of this algorithm is that there is no need to determine time moments and also no need to solve Pade equations. The reduced order model retain the initial time moments of the full system.

2.10. S. Mukherjee and R. N. Mishra approach on model order reduction of linear systems using Error minimization technique (1987) [10]:

This method is basically applicable to linear continuous systems with real distinct eigenvalues. In this approach, steady state parts of the unit-step responses of the original and reduced-order models are matched. Further, zeros are determined by minimizing the error between the transient responses. This method is not applicable if the original system has complex conjugate eigenvalues.

2.11 Jayant Pal and L. M. Ray approach of model reduction for multivariable system [11]:

This method is basically combination of Pade approximation method and Routh-Hurwitz array. This method is applicable to multivariable system. The main advantage of this method is that there is no need to determine the poles of the original system and does not involve matrix inversion.

2.12. Model order reduction by Differentiation method (1982) [12]:

This method is proposed by P. Gutman *et al.*. In this method, model order reduction is based on the differentiation of polynomials. In this approach, reciprocals of the numerator and denominator polynomials of the high-order transfer function are differentiated suitably many times to get the coefficients of the reduced order transfer functions. It is observed that this method is simple to understand.

2.13. Cauer Methods:

First of all, idea of continued fraction expansion(CFE) is reported in [13] . Using the concept of continued fraction expansion (CFE) , Chen and Shieh has first introduced the concept of model order reduction for linear time invariant system. The beauty of this technique is that there is no need to calculate eigenvalues and eigen vectors and it also retained the essential features of the system. Based on this, modified results on model order reduction have been reported

2.14. Mixed methods:

(i) Stability equation method and Modified Cauer Continued Fraction approach (1982)[14] :

This method has been proposed by R. Parthasarathy and K. N. Jayasimha approach. In this method, stable reduced order model is determined by combining advantages of the stability equation method and modified Cauer continued fraction. It is shown that the method is simple.

(ii) Stable reduced-order Pade approximants using the Routh-Hurwitz arrary (1979)[

This method has been proposed by J. Pal. In this paper, it shown that the conventional Pade approach gives unstable reduced order model if the original system is stable. The instability is mainly due to the poles of the denominator which are lying on the right-half of the s-plane. In this approach, poles of the original model will be shifted to right-half of the s-plane in the reduced model. In this paper, it shown that for such type of problems, reduced order model of the denominator can be determined by Routh criterion and reduced order model of numerator is determined by Pade approximation method.

(iii) Reduced order modelling of linear dynamic systems using Eigen spectrum analysis and modified Cauer Continued fraction(2008)[15]:

This paper has been proposed by G. Parmar and Manisha Bhandari. In this method, order reduction of linear dynamic system with real distinct eigen values can be determined by Eigen spectrum analysis and modified Cauer continued fraction method. The advantage of this method is that if the original model is stable, then the reduced order model is always stable. The authors have extended this approach for multivariable systems.

(iv) *Order reduction of linear dynamic systems using stability equation method and GA (2007)[16]:*

This approach is proposed by G. Parmar. In this approach, order reduction of linear dynamic systems is carried out by combining the advantages of stability equation method and the error minimization of Genetic algorithm. In this approach, denominator of the reduced order model is obtained by the stability equation method and numerator terms of the lower-model transfer function is determined by minimizing the integral square error between the transient responses of original and the reduced order models using Genetic algorithm approach.

2.15. Model order reduction of unstable system (1993)[17]:

This approach is proposed by J. Yang *et. al*. In this aproach the model reduction of unstable system can be carried out such that the translation information's in the s-plane preserve the input-out properties of the system. Using translations information in the s-plane, it is possible to modify the stability of the system without losing input-output information. However, balancing needs that the model is to be asymptotically stable. This requirement will be satisfied by computing controllability and observability gramians.

3. CONCLUSION

There is no single technique, of model order reduction that, is applicable for all Class of system. Therefore, for some class of system if the original system is Stable, reduced order model is also stable by using groups of methods. But by using Pade approximation method reduced model may be unstable even if the original system is stable.

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